Math 43 Midterm 1 Review

HYPERBOLICS

REFER TO THE HYPERBOLIC FUNCTIONS SUPPLEMENT

POLAR

[1] Remember that a single point in the plane has infinitely many polar co-ordinates.
Consider the point with polar co-ordinates $(7, \frac{2\pi}{3})$.

[a] Find another pair of polar co-ordinates for this point, using a positive $r-$value, and a positive $\theta-$value.
[b] Find another pair of polar co-ordinates for this point, using a positive $r-$value, and a negative $\theta-$value.
[c] Find another pair of polar co-ordinates for this point, using a negative $r-$value, and a positive $\theta-$value.
[d] Find another pair of polar co-ordinates for this point, using a negative $r-$value, and a negative $\theta-$value.

[2] Convert the following points or equations.

[a] the point with polar co-ordinates $(8, \frac{5\pi}{6})$ to rectangular co-ordinates
[b] the point with rectangular co-ordinates $(-6, -2)$ to polar co-ordinates
[c] the rectangular equation $x^2 - y^2 - 2x = 0$ to polar
[d] the polar equation $r = \dfrac{7}{4 - 2\cos\theta}$ to rectangular
[e] the rectangular equation $3x - 2y + 6 = 0$ to polar
[f] the polar equation $r = \cos 2\theta$ to rectangular
[g] the rectangular equation $x^2 + 6y - 9 = 0$ to polar
[h] the polar equation $\theta = \dfrac{5\pi}{6}$ to rectangular

[3] Run the standard tests for symmetry for the polar equation $r^3 = 1 - \sin 2\theta$, and state the conclusions.
What is the minimum interval of $\theta-$values that must be plotted before using symmetry to complete the graph ?

[4] Find the values of $\theta \in [0, 2\pi)$ at which the graph of the polar equation $r = 2\cos 2\theta + 1$ passes through the pole.

[5] Name the shape of the graphs of the following polar equations.
If the graph is a rose curve, state the number of petals.

[a] $r = 5 - 5\sin\theta$ [b] $r = 7\sin 6\theta$ [c] $r = 5$ [d] $\theta = 7$
[e] $r = 4\sin 9\theta$ [f] $r = 2 + 3\cos\theta$ [g] $r = 6 - 4\cos\theta$ [h] $r = 3\sin\theta$
[i] $r = 6 + 2\sin\theta$

[6] Sketch the graphs of the polar equations in [5][a], [f], [g] and [i] using the shortcut process shown in lecture.
Find all $x-$ and $y-$intercepts.

[7] Determine if each polar equation corresponds to a circle, a parabola, an ellipse or a hyperbola.
If the equation corresponds to a circle, find its center & radius.
If the equation corresponds to a parabola, find its eccentricity, focus, directrix & vertex.
If the equation corresponds to an ellipse, find its eccentricity, foci, directrix, center & the endpoints of the major axes and latera recta.
If the equation corresponds to a hyperbola, find its eccentricity, foci, directrix, center, vertices & the endpoints of the latera recta.
Do not convert the equations to rectangular co-ordinates.
Final answers must be in rectangular co-ordinates.

[a] $r = \dfrac{10}{3 - 3\sin\theta}$ [b] $r = \dfrac{10}{3 - 2\cos\theta}$ [c] $r = \dfrac{10}{2 + 3\sin\theta}$ [d] $r = 10$

[8] Find the polar equations of the following conics with their focus at the pole.

[a] Parabola: directrix $x = 7$
[b] Parabola: vertex $(7, \frac{3\pi}{2})$
[c] Ellipse: eccentricity $\frac{3}{4}$, directrix $y = 5$
[d] Ellipse: vertices $(4, 0)$ and $(2, \pi)$
[e] Hyperbola: eccentricity $\frac{5}{2}$, directrix $x = -3$
[f] Hyperbola: vertices $(3, \frac{3\pi}{2})$ and $(15, \frac{3\pi}{2})$

[9] Draw diagrams and write algebraic equations involving distances to answer the following questions.

A drinking fountain is 15 feet from the wall of a school building.
[a] A cat is running on the school grounds, so that it is always three times as far from the wall as it is from the fountain. What is the shape of the cat's path ?
[b] A dog is running on the school grounds, so that it is always three times as far from the fountain as it is from the wall. What is the shape of the dog's path ?
[c] A chicken is running on the school grounds, so that it is always as far from the wall as it is from the fountain. What is the shape of the chicken's path ?

PARAMETRIC

[10] Eliminate the parameter to find rectangular equations corresponding to the following parametric equations.
For [a][d][e], write y as a function of x.

[a] $x = \dfrac{t}{1-t}$, $y = \dfrac{t-1}{1+t}$

[b] $x = 3 + 5\tan t$, $y = 4 + 2\sec t$

[c] $x = 8 + 6\cos t$, $y = 7 - \sin t$

[d] $x = 5\ln 4t$, $y = 2t^3$

[e] $x = e^{3t}$, $y = e^{-t}$

[f] $x = \cos 2t$, $y = 2\cos t$

[11] AJ is standing 24 feet from BJ, who is 5 feet tall. AJ throws a football at 30 feet per second in BJ's direction, at an angle of 60° with the horizontal, from an initial height of 6 feet.

[a] Write parametric equations for the position of the football.
[b] Does the football hit BJ, go over BJ's head, or hit the ground before reaching BJ ?

[12] Find parametric equations for the following curves using templates from your lecture notes, textbook and exercises.

[a] the line through $(-3, -6)$ and $(7, -2)$
[b] the circle with $(-3, -6)$ and $(7, -2)$ as endpoints of a diameter
[c] the portion of the graph of $y = 2x^4 - 3x^3 + 1$ from $(-1, 6)$ to $(2, 9)$

[13] Without graphing (or using your calculator), describe the difference between the curves with parametric equations
$x = 1 - t^4$, $y = t^4$, $\quad x = 1 - e^t$, $y = e^t$, $\quad x = 1 - \ln t$, $y = \ln t$, and $x = 1 - \sin t$, $y = \sin t$.

ANSWERS

POLAR

[1] [a] $(7, \frac{8\pi}{3})$ [b] $(7, -\frac{4\pi}{3})$ [c] $(-7, \frac{5\pi}{3})$ [d] $(-7, -\frac{\pi}{3})$

[2] [a] $(-4\sqrt{3}, 4)$ [b] $(2\sqrt{10}, 3.46)$

[c] $r = \dfrac{2\cos\theta}{\cos 2\theta} = 2\cos\theta\sec 2\theta$ [d] $12x^2 + 16y^2 - 28x - 49 = 0$

[e] $r = \dfrac{6}{2\sin\theta - 3\cos\theta}$ [f] $(x^2 + y^2)^3 = (x^2 - y^2)^2$

[g] $r = \dfrac{3}{1+\sin\theta}$ *or* $\dfrac{-3}{1-\sin\theta}$ [h] $y = -\frac{\sqrt{3}}{3}x$

[3] Symmetry over polar axis: substituting $(r, -\theta)$ gives $r^3 = 1 + \sin 2\theta$ no conclusion

substituting $(-r, \pi - \theta)$ gives $r^3 = -1 - \sin 2\theta$ no conclusion

Symmetry over pole: substituting $(-r, \theta)$ gives $r^3 = -1 + \sin 2\theta$ no conclusion

substituting $(r, \pi + \theta)$ gives $r^3 = 1 - \sin 2\theta$ symmetric over pole

Symmetry over $\theta = \frac{\pi}{2}$: substituting $(-r, -\theta)$ gives $r^3 = -1 - \sin 2\theta$ no conclusion

substituting $(r, \pi - \theta)$ gives $r^3 = 1 + \sin 2\theta$ no conclusion

Minimum interval $\theta \in [0, \pi]$ or $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

[4] $0 = 2\cos 2\theta + 1$ for $0 \le \theta < 2\pi$

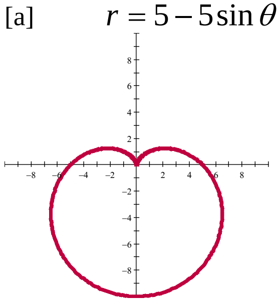
$\cos 2\theta = -\frac{1}{2}$

$2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$ since $0 \le 2\theta < 4\pi$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

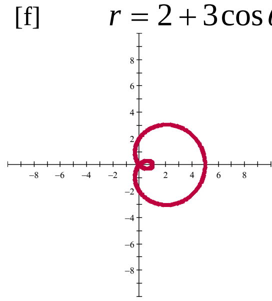
[5] [a] cardioid [b] rose curve with 12 petals [c] circle [d] line
[e] rose curve with 9 petals [f] limacon with inner loop [g] limacon with dimple [h] circle
[i] convex limacon

[6] [a] $r = 5 - 5\sin\theta$



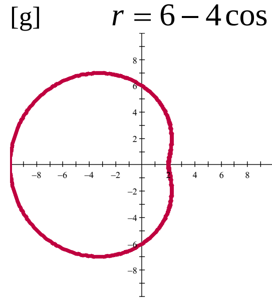
$(\pm 5, 0)$, $(0, 0)$, $(0, -10)$

[f] $r = 2 + 3\cos\theta$



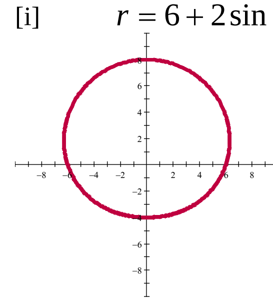
$(5, 0)$, $(1, 0)$, $(0, 0)$, $(0, \pm 2)$

[g] $r = 6 - 4\cos\theta$



$(2, 0)$, $(-10, 0)$, $(0, \pm 6)$

[i] $r = 6 + 2\sin\theta$



$(\pm 6, 0)$, $(0, 8)$, $(0, -4)$

[7] [a] PARABOLA

Eccentricity: 1

Focus: $(0, 0)$

Directrix: $y = -\frac{10}{3}$

Vertex: $(0, -\frac{5}{3})$

[b] ELLIPSE

Eccentricity: $\frac{2}{3}$

Foci: $(0, 0)$ and $(8, 0)$

Directrix: $x = -5$

Center: $(4, 0)$

Endpoints of major axis: $(-2, 0)$ and $(10, 0)$

Endpoints of latera recta: $(0, \pm\frac{10}{3})$ and $(8, \pm\frac{10}{3})$

[c] HYPERBOLA

Eccentricity: $\frac{3}{2}$

Foci: $(0, 0)$ and $(0, 12)$

Directrix: $y = \frac{10}{3}$

Center: $(0, 6)$

Vertices: $(0, 2)$ and $(0, 10)$

Endpoints of latera recta: $(\pm 5, 0)$ and $(\pm 5, 12)$

[d] Center: $(0, 0)$

Radius: 10

[8] [a] $r = \dfrac{7}{1+\cos\theta}$ [b] $r = \dfrac{14}{1-\sin\theta}$ [c] $r = \dfrac{15}{4+3\sin\theta}$

[d] $r = \dfrac{8}{3-\cos\theta}$ [e] $r = \dfrac{15}{2-5\cos\theta}$ [f] $r = \dfrac{15}{2-3\sin\theta}$

[9] [a] ellipse [b] (part of) hyperbola [c] parabola

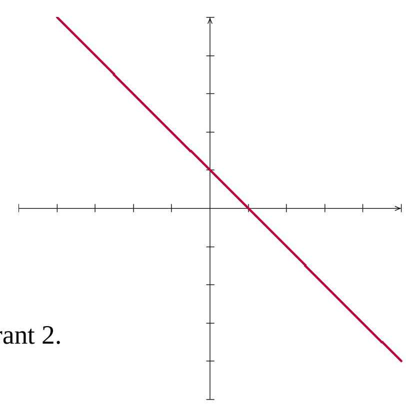
[10] [a] $y = \dfrac{-1}{2x+1}$ [b] $\dfrac{(y-4)^2}{4} - \dfrac{(x-3)^2}{25} = 1$ [c] $\dfrac{(x-8)^2}{36} + (y-7)^2 = 1$

[d] $y = \dfrac{1}{32} e^{\frac{3x}{5}}$ [e] $y = x^{-\frac{1}{3}}$ [f] $x = \dfrac{y^2}{2} - 1$

[11] [a] $x = 15t$, $y = 6 + 15\sqrt{3}\,t - 16t^2$ [b] over BJ's head

[12] [a] $x = -3 + 10t$, $y = -6 + 4t$ [b] $x = 2 + \sqrt{29}\cos t$, $y = -4 + \sqrt{29}\sin t$ [c] $x = t$, $y = 2t^4 - 3t^3 + 1$, $t \in [-1, 2]$

[13] All the parametric equations correspond to the line $x = 1 - y$ or $y = 1 - x$



Parametric equations 1:

As t goes from $-\infty$ to ∞, $y = t^4$ goes from ∞ to 0 to ∞.
The parametric curve starts in the upper left side of quadrant 2, goes to the $x-$axis, then goes back to the upper left side of quadrant 2.

Parametric equations 2:

As t goes from $-\infty$ to ∞, $y = e^t$ goes from 0 to ∞.
The parametric curve starts near the $x-$axis, then goes to the upper left side of quadrant 2.

Parametric equations 3:

As t goes from 0 to ∞, $y = \ln t$ goes from $-\infty$ to ∞.
The parametric curve starts in the lower right side of quadrant 4, then goes through quadrant 1 to the upper left side of quadrant 2.

Parametric equations 4:

As t goes from $-\infty$ to ∞, $y = \sin t$ goes back and forth between -1 and 1.
The parametric curve goes back and forth between the points $(2, -1)$ and $(0, 1)$.